Vector Algebra

Question1

If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then the value of λ is

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Options:

A. 1

B. ± 1

C. -1

D. 0

Answer: C

Solution:

Given:

$\vec{a} = \hat{i} + 2\hat{j} + \hat{k} \quad \text{and} \quad \vec{b} = \hat{i} - \hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

We need the expression $\vec{a} + \lambda\vec{b}$ to be perpendicular to $\vec{c}$:

$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

First, calculate $\vec{a} + \lambda\vec{b}$:

$\vec{a} + \lambda\vec{b} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$

Expanding this, we get:

$= \hat{i} + 2\hat{j} + \hat{k} + \lambda\hat{i} - \lambda\hat{j} + 4\lambda\hat{k}$

$= (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (1+4\lambda)\hat{k}$

Next, calculate the dot product with $\vec{c}$:

$((1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (1+4\lambda)\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$

Performing the dot product:

$= (1+\lambda)\cdot 1 + (2-\lambda)\cdot 1 + (1+4\lambda)\cdot 1$

$= (1+\lambda) + (2-\lambda) + (1+4\lambda)$

$= 1+\lambda+2-\lambda+1+4\lambda$

$= 4+4\lambda$

Set the expression to zero for perpendicularity:

$4+4\lambda = 0$

Solving for λ:

$4\lambda = -4$

$\lambda = -1$

Thus, the correct value of λ is -1.

Question2

If $|\overrightarrow{\mathrm{a}}| = 10, |\overrightarrow{\mathrm{b}}| = 2$ and $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}} = 12$, then the value of $|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$ is

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Options:

A. 5

B. 10

C. 14

D. 16

Answer: D

Solution:

To find the value of $|\overrightarrow{\mathrm{a}} \times \overrightarrow{\mathrm{b}}|$, remember that the relationship with the dot product and magnitudes is given by:

$|\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{a}||\overrightarrow{b}| \sin\theta$

where θ is the angle between the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$.

We use the formula for the dot product to calculate $\cos\theta$:

$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}| \cos\theta$

Given:

$|\overrightarrow{a}| = 10$

$|\overrightarrow{b}| = 2$

$\overrightarrow{a} \cdot \overrightarrow{b} = 12$

Substitute these values into the dot product formula:

$12 = 10 \times 2 \times \cos\theta$

Thus,

$12 = 20\cos\theta \implies \cos\theta = \frac{12}{20} = \frac{3}{5}$

Since $\sin^2\theta + \cos^2\theta = 1$, we find $\sin\theta$:

$\sin^2\theta = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$

$\sin\theta = \frac{4}{5}$

Now, substitute back to find the magnitude of the cross product:

$|\overrightarrow{a} \times \overrightarrow{b}| = 10 \times 2 \times \frac{4}{5} = 20 \times \frac{4}{5} = 16$

Therefore, the value of $|\overrightarrow{a} \times \overrightarrow{b}|$ is 16.

Question3

Consider the following statements :

Statement (I) : If either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, then $\vec{a} \cdot \vec{b} = 0$

Statement (II) : If $\vec{a} \times \vec{b} = \overrightarrow{0}$, then a is perpendicular to b. Which of the following is correct?

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Options:

A. Statement (I) is true but Statement (II) is false

B. Statement (I) is false but Statement (II) is true

C. Both Statement (I) and Statement (II) is true

D. Both Statement (I) and Statement (II) is false

Answer: A

Solution:

Let's check each statement in turn:

Statement (I): "If either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, then $\vec{a} \cdot \vec{b} = 0$."

– If one of the vectors is the zero vector, say $\vec{a} = \vec{0}$, then for any $\vec{b}$

$\vec{0} \cdot \vec{b} = 0$.

– Hence (I) is **true**.

Statement (II): "If $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a}$ is perpendicular to $\vec{b}$."

– Recall $\|\vec{a} \times \vec{b}\| = |\vec{a}|\ |\vec{b}|\ \sin\theta$, where θ is the angle between them.

– $\vec{a} \times \vec{b} = \vec{0}$ means $\sin\theta = 0$, so $\theta = 0$ or π: the vectors are **parallel** (or one is zero), not perpendicular.

– Hence (II) is **false**.

Conclusion:

Statement (I) is true, Statement (II) is false.

Question4

The vectors $\mathbf{AB} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ and $\mathbf{AC} = 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ are the sides of a $\triangle ABC$, The length of the median through A is

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Options:

A. $\sqrt{18}$

B. $\sqrt{72}$

C. $\sqrt{33}$

D. $\sqrt{288}$

Answer: C

Solution:

Let A $=$ origin

$\therefore \mathbf{AB} =$ Position vector of $\mathbf{B}$

$\mathbf{AC} =$ Position vector of $\mathbf{C}$

$\therefore$ Position vector of mid-point of $\mathbf{B}$ and $\mathbf{C}$

$$= \mathbf{P} = \frac{\mathbf{AB} + \mathbf{AC}}{2}$$
$$= \left(\frac{3+5}{2}\right)\hat{\mathbf{i}} + \left(\frac{0-2}{2}\right)\hat{\mathbf{j}} + \left(\frac{4+4}{2}\right)\hat{\mathbf{k}}$$
$$= 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

Median $= \mathbf{AP} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Hence, length of median $= |\mathbf{AP}| = \sqrt{16+1+16}$

$= \sqrt{33}$

Question5

The volume of the parallelopiped whose co terminous edges are $\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is

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Options:

A. 6 cu units

B. 2 cu units

C. 4 cu units

D. 3 cu units

Answer: B

Solution:

The volume of the parallelopiped whose coterminous edges are $\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+\hat{\mathbf{k}}$ and

$$\begin{aligned} \hat{\mathbf{i}}+\hat{\mathbf{j}} &= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} \\ &= 0-1(0-1)+1(1-0) \\ &= -1(-1)+1 \\ &= 2 \text{ cu units} \end{aligned}$$

Question6

Let a and b be two unit vectors and θ is the angle between them. Then, a + b is a unit vector, if

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Options:

A. $\theta=\frac{\pi}{4}$

B. $\theta=\frac{\pi}{3}$

C. $\theta=\frac{2\pi}{3}$

D. $\theta=\frac{\pi}{2}$

Answer: C

Solution:

$\because a+b$ is a unit vector if $|a+b|=1$

$$\begin{aligned}
\Rightarrow \quad & |\mathbf{a}+\mathbf{b}|^2=1 \\
\Rightarrow \quad & (\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})=1 \\
\Rightarrow \quad & |\mathbf{a}|^2+|\mathbf{b}|^2+2(\mathbf{a}\cdot\mathbf{b})=1 \\
\Rightarrow \quad & 1+1+2(\mathbf{a}\cdot\mathbf{b})=1 \\
\therefore \quad & 2(\mathbf{a}\cdot\mathbf{b})=-1 \\
\Rightarrow \quad & \mathbf{a}\cdot\mathbf{b}=-\tfrac{1}{2} \\
\Rightarrow \quad & |\mathbf{a}||\mathbf{b}|\cos\theta=-\tfrac{1}{2} \\
\Rightarrow \quad & 1\times 1\times\cos\theta=-\tfrac{1}{2}
\end{aligned}$$

$$\Rightarrow \quad \cos\theta=-\frac{1}{2}$$

$$\text{Hence,} \quad \theta=\frac{2\pi}{3}$$

Question7

If a, b and c are three non-coplanar vectors and p, q and r are vectors defined by $\mathbf{p}=\frac{\mathbf{a}\times\mathbf{c}}{[\mathbf{abc}]}, \mathbf{q}=\frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{abcb}}, \mathbf{r}=\frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{ab}]}$, then $(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}+(\mathbf{b}+\mathbf{c})\cdot\mathbf{q}+(\mathbf{c}+\mathbf{a})\cdot\mathbf{r}$ is

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Options:

A. 0

B. 1

C. 2

D. 3

Answer: D

Solution:

$\mathbf{p}=\frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{abc}]}, \mathbf{q}=\frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{abc}]}, \mathbf{r}=\frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{abc}]}$

Now, $(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}+(\mathbf{b}+\mathbf{c})\cdot\mathbf{q}+(\mathbf{c}+\mathbf{a})\cdot\mathbf{r}$

$$=\frac{a\cdot(b\times a)}{[abc]}+\frac{b\cdot(b\times c)}{[abc]}+\frac{b\cdot(c\times a)}{[abc]}$$
$$+\frac{c\cdot(c\times a)}{[abc]}+\frac{c\cdot(a\times b)}{[abc]}+\frac{a\cdot(a\times b)}{[abc]}$$
$$=\frac{[abc]}{[abc]}+0+\frac{[bca]}{[abc]}+0+\frac{[cab]}{[abc]}+0$$

[$\because$ scalar triple product is zero if two vectors are same]

$=\frac{[abc]}{[abc]}+\frac{[abc]}{[abc]}+\frac{[abc]}{[abc]}=3$

Question8

$|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2=144$ and $|\mathbf{a}|=4$, then $|\mathbf{b}|$ is equal to

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Options:

A. 3

B. 8

C. 4

D. 12

Answer: A

Solution:

Given, $|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2=144$ and $|\mathbf{a}|=4$

$\Rightarrow |\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta+|\mathbf{a}||\mathbf{b}|^2\cos^2\theta=144$

$\Rightarrow \quad |\mathbf{a}|^2|\mathbf{b}|^2\left[\sin^2\theta+\cos^2\theta\right]=144$

$\Rightarrow \quad (4)^2\times|\mathbf{b}|^2=144$

$\Rightarrow \quad |\mathbf{b}|^2=\frac{144}{16}=9$

$|\mathbf{b}|=3$

Question9

If $\mathbf{a}+2\mathbf{b}+3\mathbf{c}=0$ and $(\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})=\lambda(\mathbf{b}\times\mathbf{c})$, then the value of λ is equal to

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Options:

A. 3

B. 4

C. 6

D. 2

Answer: C

Solution:

Given, $\mathbf{a}+2\mathbf{b}+3\mathbf{c}=0$

$$\begin{aligned}
& (\mathbf{a}+2\mathbf{b}+3\mathbf{c})\times\mathbf{c}=0 \Rightarrow \mathbf{c}\times\mathbf{a}=2(\mathbf{b}\times\mathbf{c}) \\
& (\mathbf{a}+2\mathbf{b}+3\mathbf{c})\times\mathbf{b}=0 \\
\Rightarrow\quad & \mathbf{a}\times\mathbf{b}=3(\mathbf{b}\times\mathbf{c}) \\
\text{So,}\quad & (\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b})=\lambda(\mathbf{b}\times\mathbf{c}) \\
& (\mathbf{b}\times\mathbf{c})+2(\mathbf{b}\times\mathbf{c})+3(\mathbf{b}\times\mathbf{c})=\lambda(\mathbf{b}\times\mathbf{c}) \\
& 6(\mathbf{b}+\mathbf{c})=\lambda(\mathbf{b}\times\mathbf{c})
\end{aligned}$$

On comparing with coefficient of $(\mathbf{b}\times\mathbf{c})$, we get $\lambda=6$

Question10

If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then

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Options:

A. $\vec{a}$ and $\vec{b}$ are parallel.

B. $\vec{a}$ and $\vec{b}$ are coincident.

C. inclined to each other at 60°.

D. $\vec{a}$ and $\vec{b}$ are perpendicular.

Answer: D

Solution:

To determine which option is correct, we can analyze the given equation using the properties of vector norms. The condition $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ implies that the vector sum and difference of vectors $\vec{a}$ and $\vec{b}$ have the same magnitude. Let's expand both sides using the formula for the magnitude of a vector sum and difference:

$|\vec{a}+\vec{b}|^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2$

And for the difference, we have:

$|\vec{a}-\vec{b}|^2=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$

Given that $|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$, we can equate the right-hand sides of the above equations:

$|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=|\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$

By simplifying this equation, we can solve for the dot product $\vec{a}\cdot\vec{b}$:

$2\vec{a}\cdot\vec{b}=-2\vec{a}\cdot\vec{b}$

$4\vec{a}\cdot\vec{b}=0$

$\vec{a}\cdot\vec{b}=0$

The dot product of two vectors is zero if and only if the vectors are orthogonal (perpendicular) to each other. Therefore, we can conclude that vectors $\vec{a}$ and $\vec{b}$ are perpendicular:

Hence, the correct answer is:

Option D

$\vec{a}$ and $\vec{b}$ are perpendicular.

Question11

The component of $\hat{\mathbf{i}}$ in the direction of the vector $\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ is

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Options:

A. 6

B. $6\sqrt{6}$

C. $\frac{\sqrt{6}}{6}$

D. $\sqrt{6}$

Answer: C

Solution:

We know that component of a in the direction of b is

$\frac{|\mathbf{a}\cdot\mathbf{b}|}{|\mathbf{b}|^2}$

Let $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{a}=\hat{\mathbf{i}}$

Then, we have $|\mathbf{a}\cdot\mathbf{b}|=1$and $|\mathbf{b}|=\sqrt{6}$

Now, the component of i in the direction $\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ is $\frac{1}{\sqrt{6}}=\frac{\sqrt{6}}{6}$

Question12

If $|\mathbf{a}|=2$ and $|\mathbf{b}|=3$ and the angle between a and b is $120°$, then the length of the vector $\left|\frac{\mathbf{a}}{2}-\frac{\mathbf{b}}{3}\right|$ is

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Options:

A. 2

B. 3

C. 1/6

D. 1

Answer: B

Solution:

We know that,

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos \theta$

$$\text{So, } \mathbf{a} \cdot \mathbf{b} = (2)(3) \cos 120^\circ \quad [\because \theta = 120^\circ]$$
$$= 6 \cos (90^\circ + 30^\circ) = 6 (-\sin 30^\circ) = -6 \times \frac{1}{2}$$

$\mathbf{a} \cdot \mathbf{b} = -3$

$$\text{Now, } \left| \frac{1}{2}\mathbf{a} - \frac{1}{3}\mathbf{b} \right|^2 = \left(\frac{1}{2}\mathbf{a} - \frac{1}{3}\mathbf{b} \right) \left(\frac{1}{2}\mathbf{a} - \frac{1}{3}\mathbf{b} \right)$$
$$= \frac{1}{4}|\mathbf{a}|^2 - \frac{1}{6}\mathbf{a} \cdot \mathbf{b} - \frac{1}{6}\mathbf{a} \cdot \mathbf{b} + \frac{1}{9}|\mathbf{b}|^2$$
$$= \frac{1}{4} \times 4 - \frac{1}{6} \times (-3) - \frac{1}{6}(-3) + \frac{1}{9} \times 9$$
$$= 1 + \frac{1}{2} + \frac{1}{2} + 1 = 2 + 1 = 3$$

Hence, option (b) is correct.

Question13

If $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 36$ and $|\mathbf{a}| = 3$, then $|\mathbf{a}|$ is equal to

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Options:

A. 9

B. 36

C. 4

D. 2

Answer: D

Solution:

Given, $|\mathbf{a} \times \mathbf{b}| + |\mathbf{a} \cdot \mathbf{b}|^2 = 36$ and $|\mathbf{a}| = 3$

Correct one is $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = 36$ and $|\mathbf{a}| = 3$

We know, $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2$

$\therefore \quad |\mathbf{a}|^2|\mathbf{b}|^2 = 36 \Rightarrow (3)^2|\mathbf{b}|^2 = 36$

$\Rightarrow \quad |b|^2 = 4 \Rightarrow |b| = 2$

Hence, the correct option is (d).

Question14

If $\alpha = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}, \beta = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, then express β in the form $\beta = \beta_1 + \beta_2$ where β_1 is parallel to α and β_2 is perpendicular to α, then β_1 is given by

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Options:

A. $\frac{-1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}$

B. $\frac{5}{8}(\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

C. $\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$

D. $\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

Answer: A

Solution:

Given $\alpha = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}, \boldsymbol{\beta} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\beta = \beta_1 + \beta_2$

β_1 is parallel to α

$$\Rightarrow \quad \beta_1 = \lambda\alpha \Rightarrow \beta_1 = \lambda\hat{\mathbf{i}} - 3\lambda\hat{\mathbf{j}}$$
$$\Rightarrow \text{ Also, } \beta = \beta_1 + \beta_2$$
$$\beta_2 = \beta - \beta_1 = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (\lambda\hat{\mathbf{i}} - 3\lambda\hat{\mathbf{j}})$$
$$\beta_2 = (1-\lambda)\hat{\mathbf{i}} + (2-3\lambda)\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

It is given β_2 is perpendicular to α.

$$\therefore (1-\lambda)(1) + (2+3\lambda)(-3) + (-1)(0) = 0$$
$$\Rightarrow \quad 1 - \lambda - 6 - 9\lambda + 0 = 0$$
$$\Rightarrow \quad -5 - 10\lambda = 0$$
$$\Rightarrow \quad \lambda = -\frac{1}{2}$$
$$\beta_1 = \frac{-1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}}$$

Question15

A vector a makes equal acute angles on the coordinate axis. Then the projection of vector b $= 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + \hat{\mathbf{k}}$ on a is

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Options:

A. $\frac{11}{15}$

B. $\frac{11}{\sqrt{3}}$

C. $\frac{4}{5}$

D. $\frac{3}{5\sqrt{3}}$

Answer: B

Solution:

Let the equal angles of a with the coordinate axis be α.

$$l = \cos\alpha, m = \cos\alpha, n = \cos\alpha$$
$$\because \quad l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad 3\cos^2\alpha = 1$$
$$\Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{3}}$$

Direction ratios are $1, 1, 1$.

$$\therefore \quad \mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Projection of $\mathbf{b}$ on $\mathbf{a} = \frac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|}$

$$= \frac{(5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{1^2 + 1^2 + 1^2}}$$
$$= \frac{5 + 7 - 1}{\sqrt{3}}$$
$$= \frac{11}{\sqrt{3}}$$

Question16

The diagonals of a parallelogram are the vectors $3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. and $-\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$. Then the length of the shorter side of parallelogram is

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Options:

A. $2\sqrt{3}$

B. $\sqrt{14}$

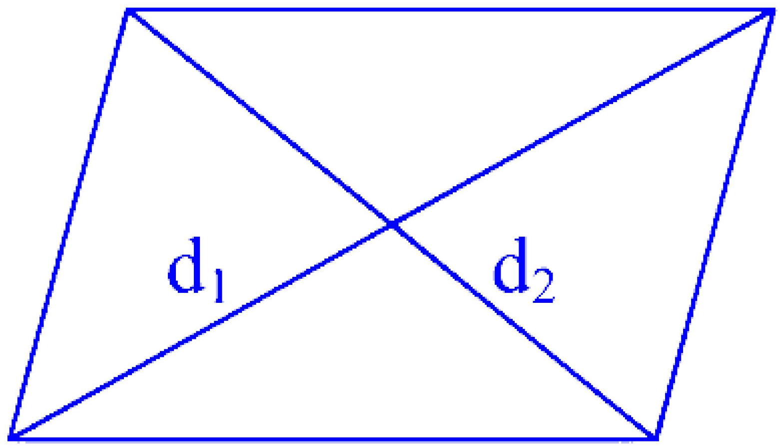
C. $3\sqrt{5}$

D. $4\sqrt{3}$

Answer: A

Solution:

Let $\mathbf{a}$ and $\mathbf{b}$ be the length of the sides of the parallelogram and $\mathbf{d}_1, \mathbf{d}_2$ be the length of diagonals.



Given, $\mathbf{d}_1 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\mathbf{d}_2 = -\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$\therefore \quad \mathbf{a} = \frac{\mathbf{d}_1 + \mathbf{d}_2}{2} = \frac{2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 10\hat{\mathbf{k}}}{2} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

$\Rightarrow \quad |\mathbf{a}| = \sqrt{1 + 4 + 25} = \sqrt{30}$

and $\quad \mathbf{b} = \frac{\mathbf{d}_1 - \mathbf{d}_2}{2} = \frac{4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{2}$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\Rightarrow \quad |\mathbf{b}| = \sqrt{4 + 16 + 9} = \sqrt{29}$

Question17

If $\mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a} + \mathbf{b}$ makes an angle 60° with a, then

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Options:

A. $|\mathbf{a}| = 2|\mathbf{b}|$

B. $2|\mathbf{a}| = |\mathbf{b}|$

C. $|\mathbf{a}| = \sqrt{3}|\mathbf{b}|$

D. $\sqrt{3}|\mathbf{a}| = |\mathbf{b}|$

Answer: D

Solution:

Given, $\mathbf{a} \cdot \mathbf{b} = 0$ and $(\mathbf{a} + \mathbf{b})$ makes 60° angle with a.

$$\cos 60^\circ = \frac{(\mathbf{a}+\mathbf{b})\cdot\mathbf{a}}{|\mathbf{a}+\mathbf{b}||\mathbf{a}|}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{|\mathbf{a}|^2+\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}+\mathbf{b}||\mathbf{a}|}$$

$$\Rightarrow \quad |\mathbf{a}+\mathbf{b}| = 2|\mathbf{a}|$$

$$\Rightarrow \quad |\mathbf{a}+\mathbf{b}|^2 = 4|\mathbf{a}|^2$$

$$\Rightarrow \quad (\mathbf{a}+\mathbf{b})(\mathbf{a}+\mathbf{b}) = 4|\mathbf{a}|^2$$

$$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b} = 4|\mathbf{a}|^2$$

$$\Rightarrow \quad \mathbf{b}^2 = 3|\mathbf{a}|^2$$

$$\Rightarrow \quad |\mathbf{b}| = \sqrt{3}|\mathbf{a}|$$

Question18

If the area of the parallelogram with a and b as two adjacent sides is 15 sq units, then the area of the parallelogram having $3\mathrm{a} + 2\mathrm{b}$ and $\mathrm{a} + 3\mathrm{b}$ as two adjacent sides in sq units is

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Options:

A. 45

B. 75

C. 105

D. 120

Answer: C

Solution:

Area of parallelogram having **a** and **b** as its adjacent sides is 15 sq units.

$\Rightarrow \quad |\mathbf{a} \times \mathbf{b}| = 15$

Area of parallelogram having $(3a + 2b)$ and $(\mathbf{a} + 3\mathbf{b})$ as two adjacent side

$$\begin{aligned} &= |(3a + 2b) \times (\mathbf{a} + 3\mathbf{b})| \\ &= |3\mathbf{a} \times \mathbf{a} + 2\mathbf{b} \times \mathbf{a} + 9\mathbf{a} \times \mathbf{b} + 6\mathbf{b} \times \mathbf{b}| \\ &= |2\mathbf{b} \times \mathbf{a} + 9\mathbf{a} \times \mathbf{b}| \end{aligned}$$

$[\because \mathbf{a} \times \mathbf{a} = \mathbf{b} \times \mathbf{b} = 0]$
$[\because \mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}]$

$= 7|\mathbf{a} \times \mathbf{b}|$
$= 7 \times 15$
$= 105$

Question19

The two vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ represent the two sides $\overline{AB}$ and $\overline{AC}$ respectively of a $\triangle ABC$. The length of the median through A is

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Options:

A. $\frac{\sqrt{14}}{2}$

B. 14

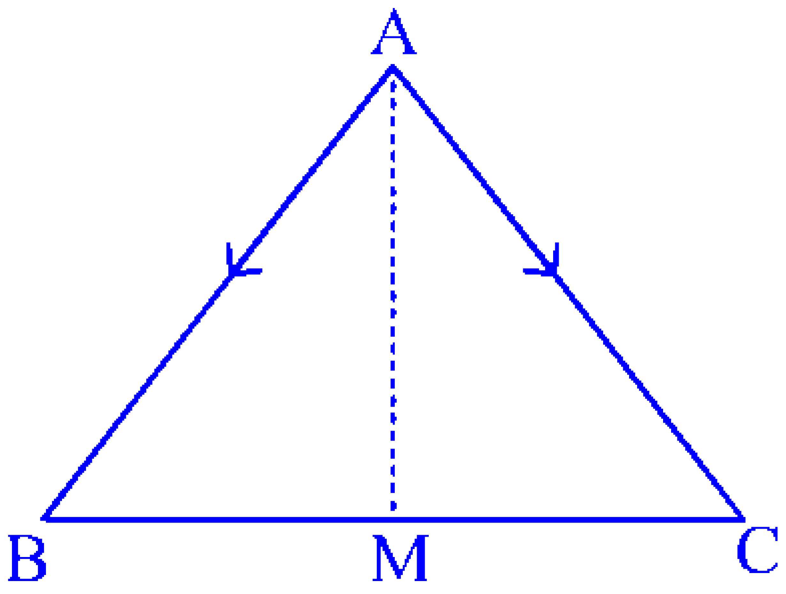
C. 7

D. $\sqrt{14}$

Answer: D

Solution:

We know that, the sum of three vectors of triangle is zero.



$\therefore \quad \mathbf{AB}+\mathbf{BC}+\mathbf{CA}=0$
$\Rightarrow \quad \mathbf{BC}=\mathbf{AC}-\mathbf{AB}$

$\Rightarrow \mathbf{BM}=\frac{\mathbf{AC}-\mathbf{AB}}{2}$ (since, M is a mid-point of BC)

And also,

$$\Rightarrow \quad \mathbf{AB}+\frac{\mathbf{AC}-\mathbf{AB}}{2}=\mathbf{AM}$$
$$\Rightarrow \quad \mathbf{AM}=\frac{\mathbf{AB}+\mathbf{AC}}{2}$$
$$\Rightarrow \quad \mathbf{AM}=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})+(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}})}{2}$$
$$\Rightarrow \quad \mathbf{AM}=\frac{2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+6\hat{\mathbf{k}}}{2}$$
$$\Rightarrow \quad \mathbf{AM}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$
$$\therefore |\mathbf{AM}|=\sqrt{(1)^2+(2)^2+(3)^2}$$
$$=\sqrt{1+4+9}=\sqrt{14}$$

Question20

If a and b are unit vectors and θ is the angle between a and b, then $\sin\frac{\theta}{2}$ is equal to

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Options:

A. $|\mathbf{a}+\mathbf{b}|$

B. $\frac{|\mathbf{a}+\mathbf{b}|}{2}$

C. $\frac{|\mathbf{a}-\mathbf{b}|}{2}$

D. $|\mathbf{a}-\mathbf{b}|$

Answer: C

Solution:

Given $\mathbf{a}$ and $\mathbf{b}$ are unit vectors

$$\begin{aligned}
&\therefore |\mathbf{a}|=1 \text{ and } |\mathbf{b}|=1\\
&\text{We know that, } |\mathbf{a}-\mathbf{b}|^2=(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})\\
&=|\mathbf{a}|^2-2\mathbf{a}\cdot\mathbf{b}+|\mathbf{b}|^2\\
&=|\mathbf{a}|^2-2|\mathbf{a}||\mathbf{b}|\cos\theta+|\mathbf{b}|^2\\
&=1-2\cdot1\cdot1\cdot\cos\theta+1\\
&=2-2\cos\theta\\
&\Rightarrow |\mathbf{a}-\mathbf{b}|^2=2(1-\cos\theta)\\
&=2\left(2\sin^2\theta/2\right)\\
&=4\sin^2\theta/2\\
&\Rightarrow \quad \sin^2\frac{\theta}{2}=\frac{1}{4}|a-b|\\
&\Rightarrow \quad \sin\frac{\theta}{2}=\frac{1}{2}|\mathbf{a}-\mathbf{b}|
\end{aligned}$$

Question21

If $|\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2=144|\mathbf{a}|=6$, then $|\mathbf{b}|$ is equal to

KCET 2020

Options:

A. 6

B. 3

C. 2

D. 4

Answer: C

Solution:

We have,

$|\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2=144$, and $|\mathbf{a}|=6$

We kniow that,

$$\begin{aligned} & |\mathbf{a}+\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2=(|\mathbf{a}||\mathbf{b}|)^2 \\ \Rightarrow\ & |\mathbf{a}|^2|\mathbf{b}|^2=144 \\ \Rightarrow\ & (6)^2|\mathbf{b}|^2=144 \\ \Rightarrow\ & |\mathbf{b}|^2=4 \\ \Rightarrow\ & |\mathbf{b}|=2 \end{aligned}$$

Question22

If the vectors $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\lambda\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ are coplanar, then the value of λ is

KCET 2020

Options:

A. 6

B. -5

C. -6

D. 5

Answer: A

Solution:

Given vectors $2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\lambda\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ are coplanar

$$\therefore \quad \begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2(2-1)+3(4+\lambda)+4(-2-\lambda)=0$$
$$\Rightarrow \quad 2+12+3\lambda-8+4\lambda=0$$
$$\Rightarrow \quad 6-\lambda=0$$
$$\Rightarrow \quad \lambda=6$$

Question23

If $|\mathbf{a}|=16, |\mathbf{b}|=4$, then $\sqrt{|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\cdot\mathbf{b}|^2}=$

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Options:

A. 16

B. 4

C. 64

D. 8

Answer: C

Solution:

Key Idea use $(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}\cdot\mathbf{b})^2=|\mathbf{a}|^2|\mathbf{b}|^2$

We have, $|a|=16, |b|=4$

$$\therefore \sqrt{|\mathbf{a}\times\mathbf{b}|^2+|\mathbf{a}\times\mathbf{b}|^2}=\sqrt{|\mathbf{a}|^2|\mathbf{b}|^2}$$
$$=|\mathbf{a}||\mathbf{b}|=16\times4=64$$

Question24

If the angle between a & b is $\frac{2\pi}{3}$ and the projection of a in the direction of b is -2 , the $|\mathbf{a}|=$

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Options:

A. 2

B. 4

C. 1

D. 3

Answer: B

Solution:

According to question, we have $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = -2$

$$\Rightarrow \frac{|\mathbf{a}||\mathbf{b}| \cos \theta}{|\mathbf{b}|} = -2;$$
$$|\mathbf{a}| \cos \theta = -2$$
$$\Rightarrow |\mathbf{a}| \cos \left(\frac{2\pi}{3}\right) = 2 \Rightarrow |\mathbf{a}| \left(-\frac{1}{2}\right) = -2 \Rightarrow |\mathbf{a}| = 4$$

Question25

A unit vector perpendicular to the plane containing the vector $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is

KCET 2019

Options:

A. $\frac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

B. $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

C. $\frac{-\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$

D. $\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$

Answer: A

Solution:

Let, $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{b}=-2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Now, $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ \mathbf{1} & 2 & 1\\ -2 & 1 & 3\end{vmatrix}$

$=\hat{\mathbf{i}}(6-1)-\hat{\mathbf{j}}(3+2+\hat{\mathbf{k}}(1+4)=5\hat{\mathbf{i}}-5\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

and $|\mathbf{a}\times\mathbf{b}|=\sqrt{(-5)^2+(-5)^2+(5)^2}=5\sqrt{3}$

We know that, the unit vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ is given by $\pm\frac{\mathbf{a}\times\mathbf{b}}{|\mathbf{a}\times\mathbf{b}|}$

$\therefore$ required unit vector perpendicular to the plane containing the given vectors

$=\frac{5\hat{\mathbf{i}}-5\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{5\sqrt{3}}=\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$

Question26

$[\mathbf{a}+2\mathbf{b}-\mathbf{c},\mathbf{a}-\mathbf{b},\mathbf{a}-\mathbf{b}-\mathbf{c}]=$

KCET 2019

Options:

A. $2[\mathbf{a},\mathbf{b},\mathbf{c}]$

B. 0

C. $3[\mathbf{a},\mathbf{b},\mathbf{c}]$

D. $[\mathbf{a},\mathbf{b},\mathbf{c}]$

Answer: C

Solution:

Key Idea: If $\mathbf{a} = a_1 l + a_2 m + a_3 n$;
$\mathbf{b} = b_1 l + b_2 m + b_3 n$
and $\mathbf{c} = c_1 l + c_2 m + c_3 n$

then $[\mathbf{abc}] = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad [l, m, n]$

$\therefore \quad [\mathbf{a} + 2\mathbf{b} - \mathbf{c}, \mathbf{a} - \mathbf{b}, \mathbf{a} - \mathbf{b} - \mathbf{c}]$

$= \begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1 \end{bmatrix} [\mathbf{a}, \mathbf{b}, \mathbf{c}] = 3[\mathbf{a}, \mathbf{b}, \mathbf{c}]$

Question27

If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then the value of $|\vec{b}|$ is

KCET 2018

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

We need to find the value of $|\vec{b}|$.

We have the identity:

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

Given that this is equal to 144, and $|\vec{a}| = 4$, we can substitute and rearrange:

$144 = (4)^2 |\vec{b}|^2$

So, we calculate:

$144 = 16|\vec{b}|^2$

Solving for $|\vec{b}|^2$, we get:

$|\vec{b}|^2 = \frac{144}{16} = 9$

Thus, $|\vec{b}| = \sqrt{9} = 3$.

Question28

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, then $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b})$ is equal to

KCET 2018

Options:

A. 5

B. 3

C. 6

D. 12

Answer: B

Solution:

Given, $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar vector

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow \quad a(bc - 1) - 1(c - 1) + 1(1 - b) = 0$$

$$\Rightarrow \quad abc - a - c + 1 + 1 - b = 0$$

$$\Rightarrow \quad abc - (a + b + c) = -2$$

Question29

If the vector $a\hat{i}+\hat{j}+\hat{k}; \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar $(a \neq b \neq c \neq 1)$, then the value of $abc-(a+b+c)$ is equal to

KCET 2018

Options:

A. 2

B. -2

C. 0

D. -1

Answer: B

Solution:

Given, $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar vector

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\Rightarrow a(bc-1)-1(c-1)+1(1-b)=0$$
$$\Rightarrow \quad abc-a-c+1+1-b=0$$
$$\Rightarrow \quad abc-(a+b+c)=-2$$

Question30

If $\vec{a}=\hat{i}+\lambda\hat{j}+2\hat{k}; \vec{b}=\mu\hat{i}+\hat{j}-\hat{k}$ are orthogonal and $|\vec{a}|=|\vec{b}|$, then (λ, μ) is equal to

KCET 2018

Options:

A. $\left(\frac{1}{4} \frac{7}{4}\right)$

B. $\left(\frac{7}{4}, \frac{1}{4}\right)$

C. $\left(\frac{1}{4}, \frac{9}{4}\right)$

D. $\left(\frac{-1}{4}, \frac{9}{4}\right)$

Answer: A

Solution:

We are given the vectors $\vec{a} = \hat{i} + \lambda\hat{j} + 2\hat{k}$ and $\vec{b} = \mu\hat{i} + \hat{j} - \hat{k}$, which are orthogonal, and we know that $|\vec{a}| = |\vec{b}|$.

Step 1: Orthogonal Vectors

When two vectors are orthogonal, their dot product is zero:

$$\vec{a} \cdot \vec{b} = 0$$

Substituting the given vectors into the dot product:

$$(\hat{i} + \lambda\hat{j} + 2\hat{k}) \cdot (\mu\hat{i} + \hat{j} - \hat{k}) = 0$$

Calculating the dot product:

$$\mu + \lambda - 2 = 0 \quad \Rightarrow \quad \mu + \lambda = 2 \quad \ldots(\text{i})$$

Step 2: Magnitude Equality

The magnitudes of the vectors are equal:

$$|\vec{a}| = |\vec{b}|$$

Therefore:

$$\sqrt{1 + \lambda^2 + 4} = \sqrt{\mu^2 + 1 + 1}$$

Squaring both sides:

$$1 + \lambda^2 + 4 = \mu^2 + 2$$

This simplifies to:

$$\mu^2 - \lambda^2 = 3$$

Using the identity for the difference of squares:

$$(\mu + \lambda)(\mu - \lambda) = 3$$

From equation (i), we have:

$$2(\mu - \lambda) = 3 \quad \Rightarrow \quad \mu - \lambda = \frac{3}{2} \quad \ldots(\text{ii})$$

Step 3: Solve Equations

Now, solve the simultaneous equations (i) and (ii):

$$\mu + \lambda = 2$$

$\mu - \lambda = \frac{3}{2}$

Adding these equations gives:

$2\mu = 2 + \frac{3}{2} = \frac{7}{2} \quad \Rightarrow \quad \mu = \frac{7}{4}$

Subtracting equation (ii) from equation (i):

$2\lambda = 2 - \frac{3}{2} = \frac{1}{2} \quad \Rightarrow \quad \lambda = \frac{1}{4}$

Thus, the values of (λ, μ) are:

$(\lambda, \mu) = \left(\frac{1}{4}, \frac{7}{4}\right)$

Question31

If a and b are unit vectors, then angle between a and b for $\sqrt{3}\mathbf{a} - \mathbf{b}$ to be unit vector is

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Options:

A. 45°

B. 60°

C. 90°

D. 30°

Answer: D

Solution:

To determine the angle between the unit vectors $\mathbf{a}$ and $\mathbf{b}$, given that $\sqrt{3}\mathbf{a} - \mathbf{b}$ is also a unit vector, we proceed as follows:

Since $\mathbf{a}$, $\mathbf{b}$, and $\sqrt{3}\mathbf{a} - \mathbf{b}$ are unit vectors, we have:

$|\mathbf{a}| = |\mathbf{b}| = |\sqrt{3}\mathbf{a} - \mathbf{b}| = 1$

Considering the condition:

$|\sqrt{3}\mathbf{a} - \mathbf{b}| = 1$

Squaring the magnitude gives:

$|\sqrt{3}\mathbf{a}-\mathbf{b}|^2=1$

Expanding this expression:

$(\sqrt{3}\mathbf{a}-\mathbf{b})\cdot(\sqrt{3}\mathbf{a}-\mathbf{b})=1$

This simplifies to:

$3|\mathbf{a}|^2+|\mathbf{b}|^2-2\sqrt{3}(\mathbf{a}\cdot\mathbf{b})=1$

Substituting the known magnitudes $|\mathbf{a}|=1$ and $|\mathbf{b}|=1$:

$3(1)^2+(1)^2-2\sqrt{3}\ \cos\theta=1$

This results in:

$3+1-2\sqrt{3}\cos\theta=1$

Solving for $\cos\theta$:

$4-2\sqrt{3}\cos\theta=1$

$2\sqrt{3}\cos\theta=3$

$\cos\theta=\frac{\sqrt{3}}{2}$

Thus, the angle θ is:

$\theta=\frac{\pi}{6}$ or 30°

Question32

If $\mathbf{a}=2\hat{\mathbf{i}}+\lambda\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ are orthogonal, then value of λ is

KCET 2017

Options:

A. $3/2$

B. 1

C. 0

D. $-5/2$

Answer: D

Solution:

We have,

$$\mathbf{a} = 2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + \hat{\mathbf{k}}$$
$$\text{and } \mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

Since, $\mathbf{a}$ and $\mathbf{b}$ are orthogonal

$\therefore \mathbf{a} \cdot \mathbf{b} = 0$

$$\Rightarrow \quad (2\hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + \mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 0$$
$$\Rightarrow \quad 2 + 2\lambda + 3 = 0$$
$$\Rightarrow \quad 2\lambda + 5 = 0$$
$$\Rightarrow \quad \lambda = -\tfrac{5}{2}$$

Question33

If a, b, c are unit vectors such that $a + b + c = 0$, then the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$ is equal to

KCET 2017

Options:

A. $3/2$

B. 1

C. 3

D. $-3/2$

Answer: D

Solution:

We have, a, b, c are unit vectors.

$\therefore |\mathbf{a}| = |\mathbf{b} = |\mathbf{c}| = 1$

Also, we have

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$$
$$\therefore \mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot 0$$

$$\Rightarrow \mathbf{a}\cdot\mathbf{a}+\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}=0$$
$$\Rightarrow |\mathbf{a}|^2+\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}=0 \qquad [\because |\mathbf{a}|=1]$$
$$\Rightarrow \mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}=-|\mathbf{a}|^2=-1 \qquad \ldots(i)$$

Similarly,

$$\mathbf{b}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{c}=-1 \qquad \ldots(ii)$$
$$and \quad \mathbf{c}\cdot\mathbf{a}+\mathbf{c}\cdot\mathbf{b}=-1 \qquad \ldots(iii)$$

On adding Eqs. (i), (ii) and (iii), we get

$$2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a})=-3$$
$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}=\frac{-3}{2}$$
